Rigidity of a Vibrated Amorphous Bi-Dimensional Packing of Grains

Corentin Coulais and Olivier Dauchot

GIT-SPEC, CEA-Saclay, URA 2464 CNRS, 91 191 Gif-sur-Yvette, France

The Jamming transition can be seen as a general phenomenon occurring whenever a dense assembly of “things” gets stuck and resists to an externally applied shear stress. The mechanical response of a vibrated amorphous bi-dimensional packing of grains close to the Jamming transition is investigated. Stress is applied to the media through a constant torque rheometer while surface fraction is tuned around the jamming transition. The rheometer turns, no matter how low is the applied torque. However, its motion is strongly intermittent and displays scale invariance, the fluctuations being maximal at the Jamming transition, where dynamical correlation length had been found to be divergent. We compare our results to previous ones obtained while dragging an intruder at constant force in the same experimental set-up.

§1. Introduction

Because food can be found in various states, kitchen is often a useful place to become aware of how differently matter can behave. Sugar cubes are solid, milk is liquid and butter is squishy. However realistic may the latter term be, it will not satisfy a physicist — should he/she be gourmet or not. Soft matter, such as emulsions, foams, and pastes behave like bona-fide solids on short timescales and as real liquids on long timescales: what is really meant by squishy is — to our sense — solid on short time scale and liquid on long time scale, namely *viscoelastic*. Obviously, the visco-elastic cross-over takes place at very different time-scale depending on the material: 10^{-12} s for water and tens of years for the pitch-drop experiment.[1)]

Are all solids actually liquid-like on large enough timescales? Is yield stress a finite time concept as nicely argued by some rheologists[2)]? How is rigidity related to the structure? General theoretical results in condensed matter physics (see e.g. 3)), argue that rigidity is a consequence of breaking translation invariance (in all directions). However it was recently demonstrated[4)] within a nucleation type argument that even crystals flow if an infinitesimal stress is applied. But, they do so in a way inherently different from ordinary liquids since their viscosity diverges for vanishing shear stress with an essential singularity. This singularity is immediately related to the way the surface tension between the deformed and the undeformed material depends on the applied stress. In 4) this surface tension was explicitly computed for a crystal. But what about amorphous materials? Is the viscosity still diverging for vanishing shear stress? In other words is there a surface tension between an amorphous phase and this same phase deformed by some applied shear?

There has been a recent surge of activity about amorphous materials as diverse as foams, emulsions, colloidal suspensions and granular media which, in apparent contradiction with the above statements, are said to jam into a rigid, disordered state where they withstand finite shear stresses before yielding.[5)] The crucial point to realize is that the jamming transition is a zero temperature transition whereas the above discussion refers to a thermally activated process. At the jamming point, frictionless, idealized soft spheres are marginally stable (isostatic) in the sense of constraint counting. As a result of this coincidence, many geometric,[6] vibrational[7] and mechanical[8),9] properties scale with distance to this jamming point in a different way from that of ordinary elastic solids. In particular a new class of low-frequency vibrational modes, which arises because the system is at the threshold of mechanical stability, provides a solid framework for further theoretical developments. However the way the zero and finite temperature situations are connected remains an active matter of debate.[10,11] Despite recent experiments which suggest that the soft mode picture also applies to brownian colloidal particles,[12,13] we are far from having clear answers to these questions.

Finally for frictional particles, marginal stability is no more simply connected to jamming[14,15] and the rheology of dense truly frictional granular media is even further from any theoretical description. In this context, our group has experimentally studied the dynamical and mechanical properties of a gently vibrated mono-layer of bi-disperse grains close to the jamming transition. Having identified a well defined jamming packing fraction for a given compression protocol, we have (i) characterized the spontaneous dynamical fluctuations,[16–18] (ii) studied the elementary excitation modes above jamming[19] and (iii) investigated the motion of a local probe driven by a constant force.[20,21] This last study has demonstrated the existence of a strong signature of the jamming transition in the response function of the material to an external applied stress. However, in such highly nonlinear regimes, one must check how much sensitive the response is to the type of mechanical solicitation. This is the main purpose of the present study, where we conduct rheological measurement at constant torque in the very same experimental situation. We will see that most of the conclusion driven from the local probe experiment also hold in the case of more classical rheometry.

The paper is organized as follows. In a first part we briefly describe the experimental set up and protocols, together with the newly built rheometer. We then review the previously obtained results and take that opportunity to discuss the behavior of rattlers, often quoted in the literature but usually not described, especially not across the transition. In the main part, we present the rheological observations and compare them with those obtained with the local probe experiment. Finally we discuss our results and provide some perspective for the near future.

§2. Experimental set-up

The experimental system (see Fig. 1) consists of 8250 cylinders with diameters $d_s = 4$ mm and $d_b = 5$ mm and height 3 mm covering an equal surface. This layer lays out on a glass plate ① which oscillates horizontally at a frequency $f = 10$ Hz and an amplitude $a = 10$ mm. The resulting acceleration drives the grains on the plate. The grains are mixed altogether and confined in a 2D cell ②, fixed in the laboratory frame, and covered by a Plexiglas board. The total surface of the cell can be finely tuned by the means of a piston ③ linked to a position controller (Microcontrole

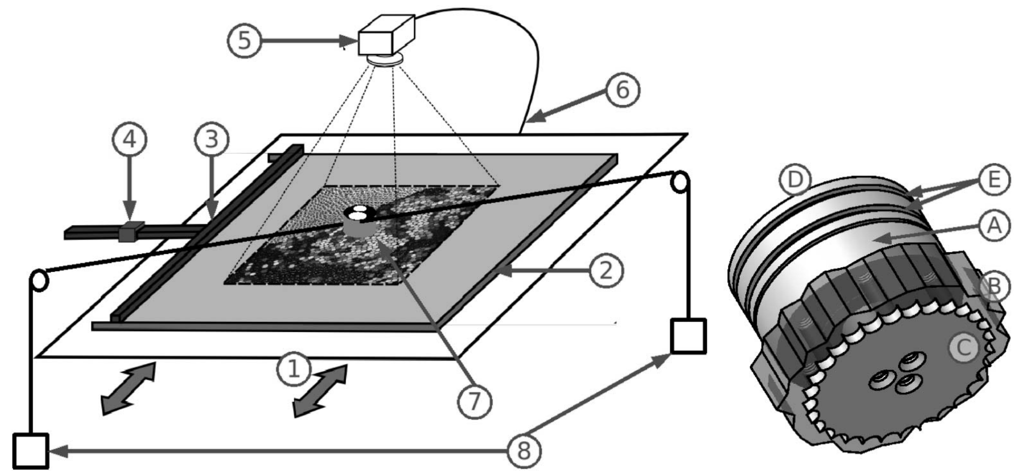
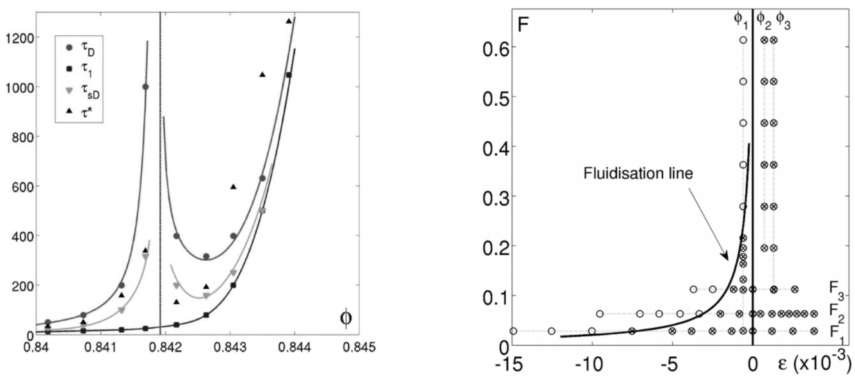


Fig. 1. Left: Original experimental setup and additional rheometer device. Right: The rheometer-wheel (see text for details).

step-motor). The 10 μm step resolution allows to tune the packing fraction fraction with increments of the order of $\delta\phi = 2.10^{-4}$. Between the controller and the piston, a force gauge ④ allows to measure the pressure at the boundary. For the local probe experiments, we have used a larger particle $d_{intruder} = 2d_s$ of the same height as the other grains pulled by a mass via a pulley perpendicularly to the vibration. In all data presented here the resultant motion is strongly over-damped and the applied force can be considered as constant. The rheometer consists in a wheel of diameter $D = 50$ mm ⑦ driven at constant torque via two symmetrical masses ⑧. The wheel Ⓒ is machined in such a way that slipping with grains remains limited. An oscillation triggered ⑥ camera ⑤ records the stroboscopic motion of the grains as well as the motion of the intruder and the angular motion of the rheometer. In the following the time unit is set to one plate oscillation while the length unit is chosen to be the diameter of the small particles d_s. The drag force F is expressed as the ratio of the pulling mass onto the total mass of grains in the cell $M_{tot} = 2.320$ kg, and the drag torque T is expressed in units of $M_{tot}D/d_s$.

§3. Protocols and control parameters

In all the experiments, the main control parameter is the packing fraction ϕ. The system is first compressed stepwise very slowly. This quench protocol produces very dense configurations such that the structural relaxation time is much larger than the experimental time scale. However the vibration still induces micro-rearrangements through collective contact slips that lead to partial exploration of the portion of phase space restricted to a particular frozen structure. For packing fractions $\phi < 0.842$ the pressure vanishes when the vibration is switched off, whereas it remains finite for larger ϕ. We have identified this behavior with jamming and shown that ϕ_J appears as a genuine critical point where a dynamical correlation length and a correlation time simultaneously diverge, showing that the dynamics occurs by involving progressively

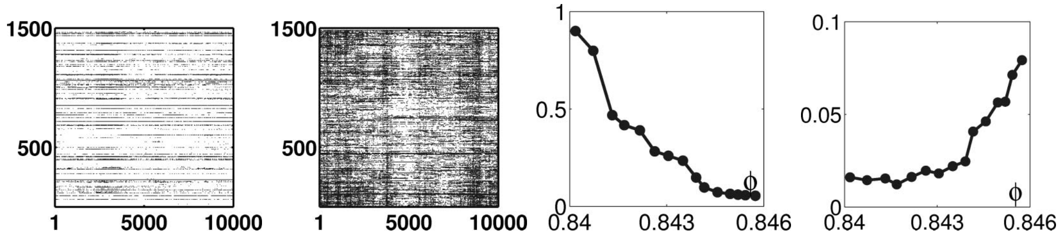


(a) (b)

Fig. 2. (a) Time-scales of the spontaneous fluctuations across jamming. At τ_1 the particles exit a sub-diffusive plateau to enter a super-diffusive regime ($\tau_s D$) which last until they recover diffusive motion (τ_D). Dynamical heterogeneities are maximal on the timescale τ^* (taken from 16)). (b) Force-packing fraction parameter space. $\epsilon = (\phi - \phi_J)/\phi_J$ is the relative distance to jamming. Each point corresponds to a trajectory of the intruder into the media. The horizontal and vertical dotted lines correspond to the experimental exploration paths, either at constant force or constant packing fraction. # (resp. $\otimes$) denote the observation of a fluidized (resp. an intermittent) motion.

more collective rearrangements (see 16) for more details). One of the most surprising result was the discovery of a super-diffusive regime while the typical displacements of the grains remains small compared to the grains diameter! This super-diffusive regime appears for time scales $\tau \sim \tau_{sD}$. It was first interpreted as the existence of large scale convective currents, which were tentatively associated to the extended soft modes that appear when the system loses or acquires rigidity at ϕ_J. Later the motion of the grains was shown to obey Levy flights, i.e. a sum of uncorrelated individual displacements with a power-law tail distribution of sizes such that the variance of the distribution diverges. This divergence only occurs at $\phi = \phi_J$, but is truncated away from the critical point, which explains why the motion reverts to normal diffusion at a very large time τ_D, which diverges at the transition (see Fig. 2). This finding shows that the rearrangements corresponding to the maximum of dynamical correlations are made of a large number of temporally incoherent jumps with a broad distribution of jum¡p sizes (see 18) for more details).

We now turn to the local probe experiments. In that case, we have either set the drag force F or the packing fraction ϕ constant as sketched in Fig. 2(b). A first "fluidization" transition separates a continuous motion regime, where the force-velocity relation is affine from an intermittent motion one, where the force-velocity relation is clearly stiffer. The force threshold increases with the packing fraction and seemingly diverges at the jamming transition. Below this threshold the intruder motion is intermittent. As we shall see below these intermittent fluctuations are maximal across the jamming transition. This evidence was supported not only by the analysis of the motion of the probe but also by that of the surrounding grains.[20),21)] The bursting events are characterized by increasingly heterogeneous patterns in the instantaneous displacement field around the intruder.



(a) (b) (c) (d)

Fig. 3. Typology of rattling particles. (a) Label of the rattling grains vs time for the highest packing fraction explored $\phi = 0.845$. (b) Same for the loosest one $\phi = 0.84$. (c) Fraction of particles which have rattled at least once vs the packing fraction. (d) Averaged fraction of time spent rattling by rattlers vs the packing fraction.

The reader will have observed that for the intruder experiment, we have used the relative distance to jamming ϵ instead of the absolute packing fraction ϕ. The reason is that the precise value of the packing fraction is extremely sensitive to the protocol. This is also the case in the thermodynamic limit,[11)] and strongly enforced by finite size effects.[6)] As a matter of fact we have recently observed that we could *control* the value of the packing fraction at which the system jams, by changing the compaction rate: the faster the compaction, the looser ϕ_J. A clear signature of the influence of the compaction rate is the density of rattlers which strongly increases with the compaction rate. Investigating the mechanical properties of such differently jammed packing is a promising path for future research. For now, let us concentrate on the slowest compaction rate. Already in that case, the typology of the rattling particles is an interesting matter (see Fig. 3): for the largest packing fraction, far into the jammed region, only a small percentage of the particles are rattlers, and they rattle frequently. When decreasing the packing fraction, the number of rattlers increases, but the average fraction of time they individually rattle decreases. Below jamming, all particles eventually rattle from time to time.

§4. Local probe versus rheometry

As underlined in the introduction, we would like to investigate how the response to a mesoscopic stress solicitation differs from the one induced by a local probe. Indeed the wheel of the rheometer covers a surface equivalent to that of a hundred grains and one may expect strong effects in such highly nonlinear response functions.

At first sight the rotational motion of the rheometer is very similar to the translational motion of the intruder (see Fig. 4(a) and compare to Fig. 3(a) of 21)). The typical velocity spans several orders of magnitude within a tiny variation of the packing fraction and the motion looks strongly intermittent, almost like steps. It can be seen as consecutive creeping (or waiting) and yielding events. While yield events display critical behavior and scaling at jamming transition,[21)] the amplitude of the averaged displacement is mainly governed by the duration of the waiting events. These durations are intimately related to the typical reorganization time scales, which were discussed in the previous section. More quantitatively, the distri-

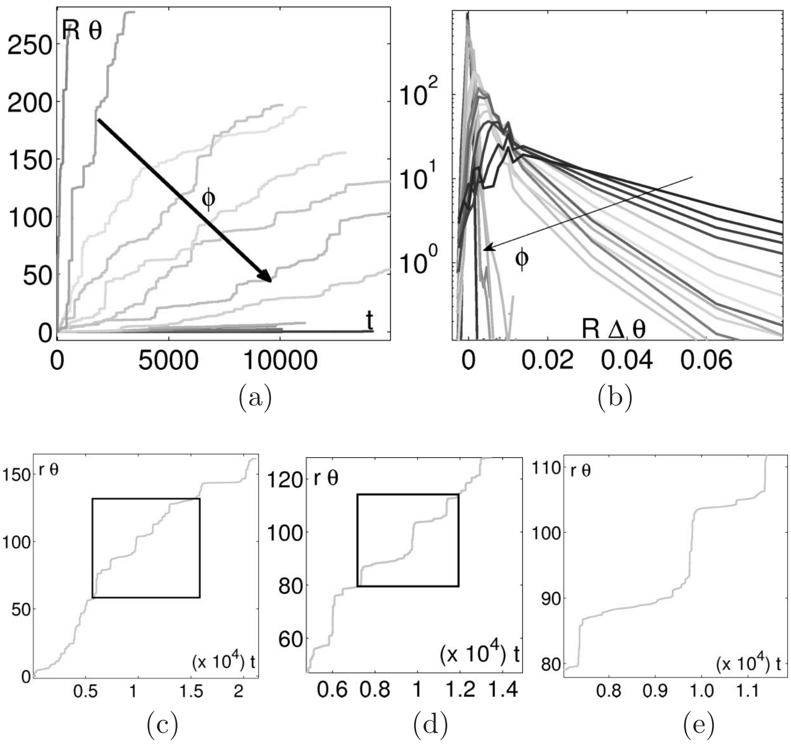


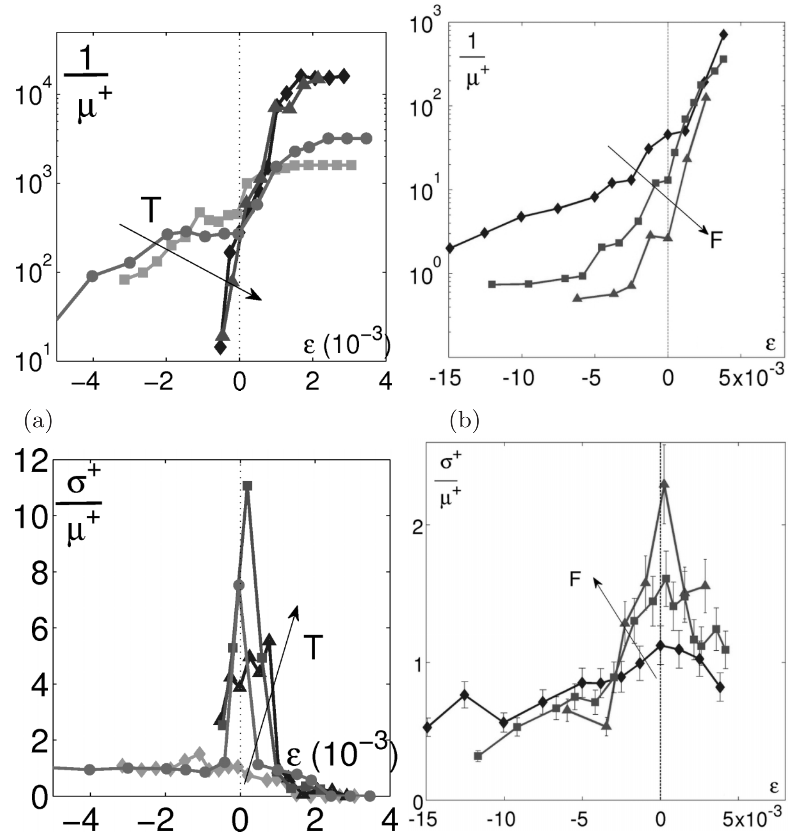
Fig. 4. (Color online) (a) Angular motion of the rheometer for several packing fractions, from $\phi = 0.8385$ (red) to $\phi = 0.8416$ (blue), at constant torque ($T = 0.138$). (b) Distribution of instantaneous displacements of the rheometer $R\Delta\theta$ for the same parameters. (c,d,e) Two levels of magnification on the angular motion of the rheometer for a surface fraction close to Jamming, and a torque $T = 0.138$.

butions of the instantaneous displacements defined as $R\Delta\theta(t) = R\theta(t+1) - R\theta(t)$, where $R = D/2$ is the rheometer wheel radius, shift towards larger rotation and exhibit larger tails as the packing fraction is reduced. Another strong similarity is that the intermittent motion of the wheel appears to be scale invariant as illustrated in Figs. 4(c), (d), and (e). In contrast, we could not observe the fluidization transition with the rheometer indicating that the grains resist collectively to the motion.

We now further characterize the rheometer dynamics when approaching the jamming transition and compare the results to those obtained with the intruder. Following 21) , we concentrate on the angular motion in the direction of the external torque $R\Delta\theta^+$. We compute the average $\mu^+ = \langle R\Delta\theta^+\rangle_t$ and the relative fluctuations:

$$\frac{\sigma^+}{\mu^+} = \frac{\sqrt{\langle(R\Delta\theta^+ - \mu^+)^2\rangle_t}}{\mu^+}. \tag{4·1}$$

The top row of Fig. 5 shows that $1/\mu^+$, the typical time the rheometer perimeter (left) and the intruder (right) take to rotate or translate over one particle diameter increases monotonically with ε and faster than exponentially. The stronger the torque the sharper the increase. In the case of the rheometer this timescale saturates because of the finite duration of the experiment. One also notes that for small enough



(a) (b) (c) (d)

Fig. 5. Inverse average $1/\mu^+$ (top) and standard deviation over average σ^+/μ^+ (bottom) of the instantaneous positive displacements in the torque direction for the rheometer (left) and in the drag direction for the intruder (right) as functions of the reduced packing fraction ε. Different curves correspond to different torques $T = 0.049$ ($\diamond$), 0.064 ($\circ$), 0.138 ($\triangle$) and 0.177 ($\square$) and drag forces $F = 0.029$ ($\diamond$), 0.064 ($\square$) and 0.113 ($\triangle$). (Graphs (b) and (c) are taken from 21)).

torques, the increase is comparable to the one obtained with the intruder, but that it becomes really much sharper for larger torque. This suggests that above ϕ_J, the torque itself contributes to the hardening of the system. In all cases no significant behavior is observed when crossing the jamming transition. On the contrary, as shown on the bottom row of Fig. 5, the transition is sharply marked by a peak of σ^+/μ^+ precisely at ϕ_J. The peak sharpens when the torque or drag force is stronger and only for the smallest torque we could not observe it. It is worth observing that the relative fluctuations are larger for the rheometer. Hence there is no self-averaging of the fluctuations among the grains driven by the rheometer. On the contrary the absolute fluctuations remain of the same order and only the average velocity is smaller in the rheometer case.

Finally, let us point out that the divergence of intermittent fluctuations at jamming as reported in Fig. 5 reminds the diverging dynamical length underscored in 16) and 17). The question of the link between the spontaneous fluctuations (that are here represented by the dynamical correlation length) and the mechanical response (that are here given by the fluctuations characterizing the intermittent motion) is a very interesting issue and a very motivating trail to investigate. There are theoretical[23] evidences that such fluctuation-dissipation theorems hold in the context of inhomogeneous mode coupling in supercooled liquids and a recent study in glycerol has provided experimental evidences of such a relation.[24] Obviously, the case of granular material close to jamming may be very specific, and some more theoretical and experimental work on this issue would be necessary in order to address it.

§5. Conclusion

Altogether, the measurements performed with the rheometer lead to the same conclusions as those conducted with the intruder. The most significant observation is that even above jamming there is motion as long as the vibration is active. However this motion is very specific. It is highly intermittent and the fluctuations are so large that one hardly imagine that they could lead to a simple visco-elastic description at the macroscopic scale. As a matter of fact, the rheometer experiment looks even more intrusive than the intruder one, suggesting that the nonlinear effects are collectively reinforced: the larger the torque, the more resistant is the system.

Similar fluctuations are reported in sheared granular materials,[25),26] and seem to intervene whenever the assembly is close to arrest. As for many other phenomena such as earthquakes,[22] Barkhausen noise,[27] crack tip[28] or elastic front[29] dynamics in heterogeneous materials that are related to the notions of jamming or depinning transitions, criticality emerges from a competition between disorder and external driving. In the present case, one would like to know how much of the disorder is related to forces versus structure.

Another interesting issue to address in the near future is the robustness of these observations with respect to the compression protocol. For faster compression the system jams at lower packing fractions. One expects these systems to be extremely more sensitive to external perturbation. How will this affect the local rheological properties? More generally, we believe that micro and macro-rheological studies combined to statistical observations such as dynamical correlations are key elements to investigate further the underlying mechanisms of the jamming transition in frictional systems. Further investigations using different driving on our system should provide fruitful clues to these challenging questions.

Acknowledgements

We would like to thank Frederic Lechenault and Raphael Candelier for their scientific advices on the experiment and the data analysis. We also thank V. Padilla and C. Gasquet for technical assistance on the experiment. We are grateful to the YKIS organizers who gave us the opportunity to present this work during the conference. This work was supported by ANR DYNHET 07-BLAN-0157-01.

References

1) http://en.wikipedia.org/wiki/Pitch_drop_experiment
2) H. A. Barnes, J. Non-Newtonian Fluid Mech. **81** (1999), 133.
3) P. M. Chaikin and T. C. Lubensky, *Principles of Condensed Matter Physics* (Cambridge University Press, 1995).
4) F. Sausset, G. Biroli and J. Kurchan, arXiv:1001.0918.
5) M. van Hecke, J. of Phys.: Cond. Mat. **22** (2010), 033101.
6) C. S. O'Hern, S. A. Langer, A. J. Liu and S. R. Nagel, Phys. Rev. Lett. **88** (2002), 075507.
C. S. O'Hern, L. E. Silbert, A. J. Liu and S. R. Nagel, Phys. Rev. E **68** (2003), 011306.
7) M. Wyart, Ann. de Phys. **30** (2005), 1.
C. Brito and M. Wyart, J. Stat. Mech. **8** (2007), L08003.
8) P. Olsson and S. Teitel, Phys. Rev. Lett. **99** (2007), 178001.
9) B. P. Tighe, E. Woldhuis, J. Remmers, W. van Saarloos and M. van Hecke, arXiv:1003.1268.
10) Z. Zhang, N. Xu, D. T. N. Chen, P. Yunker, A. M. Alsayed, K. B. Aptowicz, P. Habdas, A. J. Liu, S. R. Nagel and A. G. Yodh, Nature **459** (2009), 230.
11) L. Berthier and T. Witten, Phys. Rev. E **80** (2009), 021502.
P. Chaudhuri, L. Berthier and S. Sastry, arXiv:0910.0364.
12) A. Ghosh, V. K. Chikkadi, P. Schall, J. Kurchan and D. Bonn, arXiv:0910.3231.
13) K. Chen, W. G. Ellenbroek, Z. Zhang, D. T. N. Chen, P. Yunker, S. Henkes, C. Brito, O. Dauchot, W. Van Saarloos, A. J. Liu and A. G. Yodh, arXiv:1003.3065.
14) E. Somfai, M. Van Hecke, W. G. Ellenbroek, K. Shundyak and W. Van Saarloos, Phys. Rev. E **75** (2007), 020301.
15) S. Henkes, M. Van Hecke and W. Van Saarloos, arXiv:0907.3451.
S. Henkes, C. Brito, O. Dauchot and W. Van Saarloos, arXiv:1001.2202.
16) F. Lechenault, O. Dauchot, G. Biroli and J.-P. Bouchaud, Europhys. Lett. **83** (2008), 46002.
17) F. Lechenault, O. Dauchot, G. Biroli and J.-P. Bouchaud, Europhys. Lett. **83** (2008), 46003.
18) F. Lechenault, R. Candelier, O. Dauchot, J.-P. Bouchaud and G. Biroli, arXiv:1001.1765v1.
19) C. Brito, O. Dauchot, G. Biroli and J.-P. Bouchaud, arXiv:1003.1529.
20) R. Candelier and O. Dauchot, Phys. Rev. Lett. **103** (2009), 128001.
21) R. Candelier and O. Dauchot, Phys. Rev. E **81** (2010), 011304.
22) B. Gutenberg and C. F. Richter, *Seismicity of the Earth and Associated Phenomena* (Princeton University Press, 1954).
23) G. Biroli, J.-P. Bouchaud, K. Miyazaki and D. R. Reichman, Phys. Rev. Lett. **97** (2006), 195701.
24) C. Crauste-Thibierge, C. Brun, F. Ladieu, D. L'Hôte, G. Biroli and J.-P. Bouchaud, arXiv:1002.0498v2.
25) R. P. Behringer, D. Bi, B. Chakraborty, S. Henkes and R. R. Hartley, Phys. Rev. Lett. **101** (2008), 268301.
26) F. Dalton, F. Farrelly, A. Petri, L. Pietronero, L. Pitolli and G. Pontuale, Phys. Rev. Lett. **95** (2005), 138001.
27) G. Bertotti and I. Mayergoyz, *The Science of Hysteresis* (Academic, Oxford, 2006), p. 181.
28) K. J. Maloy, S. Santucci, J. Schmittbuhl and R. Toussaint, Phys. Rev. Lett. **96** (2006), 045501.
29) R. Planet, S. Santucci and J. Ortín, Phys. Rev. Lett. **102** (2009), 094502.